



Cambridge International AS & A Level

CANDIDATE
NAME

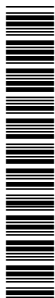
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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

3

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - i| \leq 1$ and $\arg(z - 1) \leq \frac{3}{4}\pi$. [4]

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

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It is given that the line intersects the y -axis at the point where $y = 1.3$.

(b) Calculate the value of A , giving your answer correct to 2 decimal places. [2]

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4 Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) dx$. [5]

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- 6 (a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

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- (b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta) d\theta = \frac{1}{2} \ln 2$. [4]

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- 7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, e)$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [7]

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8 The equation of a curve is $y = e^{-5x} \tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x -coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

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9 Let $f(x) = \frac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

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- (b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

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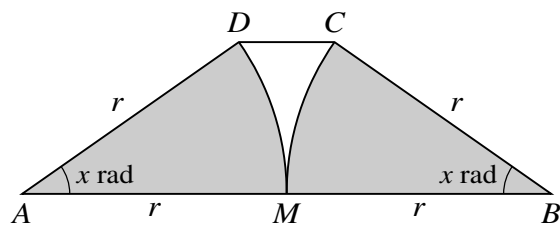
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The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M , the midpoint of AB .

- (a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x) \sin x$. [3]

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- (b) Verify by calculation that x lies between 0.5 and 0.7. [2]

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- (c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(2 - \frac{x_n}{0.9 \sin x_n}\right)$$

converges, then it converges to the root of the equation in part (a). [2]

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- (d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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11 With respect to the origin O , the points A and B have position vectors given by $\vec{OA} = 2\mathbf{i} - \mathbf{j}$ and $\vec{OB} = \mathbf{j} - 2\mathbf{k}$.

(a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

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The midpoint of AB is M . The point P on the line through O and M is such that $PA : OA = \sqrt{7} : 1$.

- (b) Find the possible position vectors of P . [6]

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Additional Page

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